Solar Household Energy, Inc.
Solar Cooking for Human Development and Environmental Relief

SHE Technical Report no. TR-01.1

Theoretical Model of Temperatures in a HotPot Solar Cooker

Paul Arveson*, Michael Dadok**

** Director of Research, Solar Household Energy*
*** George Mason University*

June 2015

Citation: Technical Report no. TR-01.1, Solar Household Energy, Inc., (June, 2015)

Solar Household Energy (SHE) strives to unleash the potential of solar cooking to improve social, economic and environmental conditions in sun-rich areas around the world. SHE Technical Reports are intended for use within the solar cooking community, for the rapid dissemination of findings related to solar cookers. They may contain information that is based on limited data, and/or conclusions and recommendations that are solely the opinions of the author, not of the organization. Please contact the author for further correspondence.

Theoretical Model of Temperatures in a HotPot Solar Cooker

Paul Arveson and Michael Dadok

1. Introduction

Many people have undertaken to design and construct solar cooking devices. This is a commendable activity, because these products have many benefits, especially in tropical regions where sunlight is plentiful but other resources are scarce or relatively costly for the typical household.

Although descriptions of cooker designs are plentiful, measurements of cooker performance are limited in the literature and on the web. A designer may include a measurement of internal temperature, but this is not a measurement of cooker power or other desired parameters. Hence it is difficult to compare different designs. What is needed is a general physical model of solar cookers that can be used to improve the performance of existing designs and compare alternatives in a controlled and meaningful way. That is the intent of this article.

The methodology here is "semi-empirical", meaning that basic physical equations, such as Newton's Law of Heating and Cooling and the Stefan-Boltzmann Law, will provide a solid physical foundation for the development of a model. However, some parameters in the model are difficult to calculate from first principles, because of complexity in shapes, materials properties and other factors. In these cases estimated values will be used based on experimental measurements. Once the model is "calibrated" with a range of cooker types, it is hoped this work will be helpful in encouraging good practices of cooker performance assessment and comparisons.

The initial model development is based on measurements of the "HotPot", a cooker developed by Solar Household Energy, Inc. and distributed in several countries around the world.

2. Developing a Model of the Heating Curve

Newton observed that the rate of change of temperature in a heating or cooling object is proportional to the temperature difference between the object and the heat source. Mathematically, for heating we can write this general observation as:

$$\frac{dQ}{dt} = K\big(M - Q(t)\big)$$

Where M is the potential temperature that can be attained due to the heat source, and $Q(t)$ is the actual temperature of the object at any time t. The inverse of K is a value called the "time constant" which indicates how fast the object's temperature can change. (The time constant is determined by the mass of the object, its thermal conductivity, surface area and other factors. It is generally determined by experiment.) This is Newton's Law of Heating and Cooling.

In our case, the heat source is the sun and *Q(t)* is the internal temperature of the cooker. The cooker tested is a 5-liter "HotPot" made by Solar Household Energy, Inc. A description of the HotPot and the experimental setup is available in the references. For these measurements, the HotPot contained 1 liter of canola cooking oil, not water. This allowed the temperature to exceed 100 deg. C without being limited by the boiling of water.

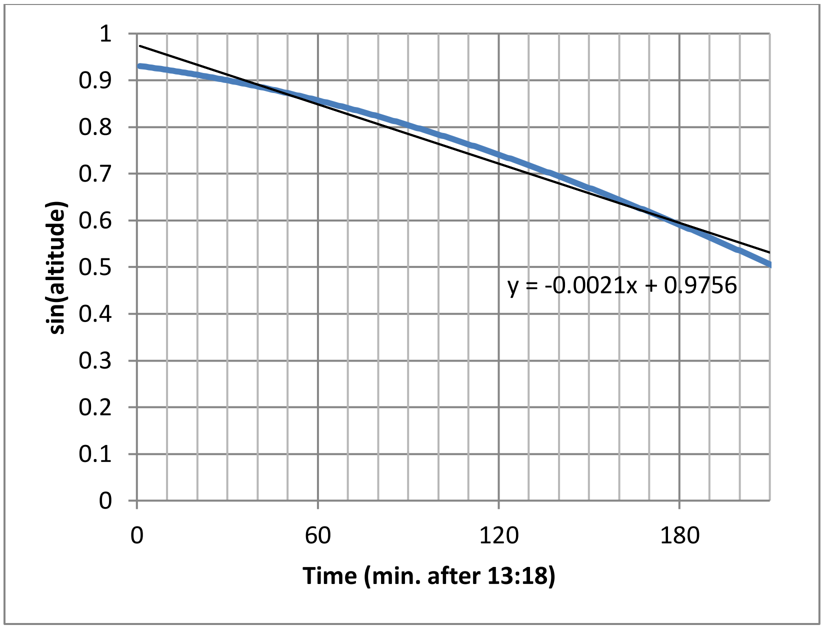
Generally solar cookers need to be aimed in order to maintain the maximum illuminated area of sunlight. Depending on the design of the particular cooker, it may be more or less sensitive to changes in the sun direction. Cookers made with parabolic reflectors are very sensitive to the sun direction, and will need to be turned more or less continuously. Panel or box cookers typically need to be turned much less frequently, or not at all. The recommendations of the cooker manufacturer should be followed for best performance. For these experiments with the HotPot panel cooker, the reflector was aimed at the angle that would maximize illuminated reflector area at noon, and left at that position.

An extension of Newton's heat law must be made in order to create a more realistic model for solar cooking. The heating power or irradiance of the sun varies as the sun moves across the sky, due to the "projection effect". Specifically, heating power varies as $\sin(\vartheta)$ where ϑ is the altitude of the sun. With this extension, the heat equation becomes:

$$\frac{dQ}{dt} = K\big(M(t) - Q(t)\big)$$

Where $M(t) = \sin(\vartheta)/\sin(\vartheta_0)$, where ϑ_0 is the sun's altitude at the beginning of temperature measurements, which began at 13:18 (1:18pm). For the day of the measurements (July 6, 2012) at the latitude of the test location (34 degrees; Rockville, MD), the altitude of the sun was obtained from the online calculator at the Naval Observatory.

Including sine functions in the differential equation will make it unnecessarily complicated. To simplify the solution, the altitude values were approximated by a linear fit to the exact values. The figure below shows the $\sin(\vartheta)$ values and the least-squares linear regression fit to these values.



The linear equation gives a reasonable approximation to the sine of the sun angle over 3 hours, which is sufficient for our purposes. Since this is a heating calculation, we know that the temperature will rise toward some equilibrium temperature *E* which is somewhat higher than the maximum internal temperature actually measured. Since the equilibrium temperature depends on the sun angle, the function defining the maximum temperature can be written as

$$M(t) = \frac{E \sin(\theta)}{\sin(\theta_0)} \approx E\ (a(t) + b)$$

Where *E* is the maximum equilibrium temperature, a = -.0021 and b = 0.9756. This linear approximation was substituted in the differential equation. This equation was solved using the Matlab symbolic solver to obtain:

$$Q(t) = M(t) - (M(t) - Q_0)e^{-Kt}$$

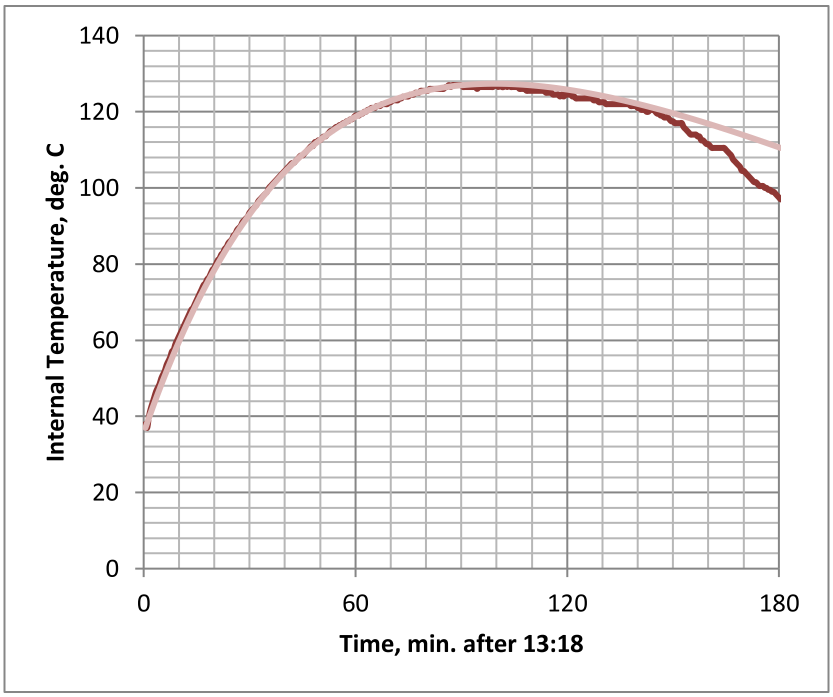
This is an initial value problem. From an examination of the data, the initial temperature Q_0 = 37 deg. C. We can solve for *K* to obtain the general formula:

$$K = -\log((E - Q_2)/t_2(E - Q_0)$$

Where Q_2 is the temperature measured at some later temperature t_2. The temperature Q_2 at 60 minutes was measured at 118 deg. C. We assume a maximum temperature *E* = 190. Given these values, we obtain:

K = -1/60*log((190-118)/(190-37)) = 0.0126 which is a time constant $1/K$ of 79 minutes. (A time constant could also be determined by measuring the cooling time of the cooker in the dark. This should be similar to the heating time constant.)

The maximum or equilibrium temperature of the cooker E was selected to give a best fit to the data. Then the final $Q(t)$ model using E = 190 and K=0.0126 gives:



The dark line is the measured data and the light line is the model.

The inclusion of the sun altitude projection effect causes the temperature curve to track the data more realistically. Decreases in the data at later times may be due to reflector shadowing later in the afternoon.

The actual temperature achieved, about 130 deg. C, is much less than the best-fit value for equilibrium temperature E. In other words, due to the loaded cooker's long time constant, the oil did not have time to reach its maximum temperature before the sun altitude decreased significantly. In these measurements, the reflector was not turned after the initial setup. Such a situation may be typical for many solar cooker designs.